

# $\Lambda(1405)$ as a multiquark state

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## Abstract

In the QCD sum rule approach we predict the  $\Lambda(1405)$  mass by choosing the  $\pi^0\Sigma^0$  multiquark interpolating field. It is found that the mass is about 1.419 GeV from  $\Pi_1(q^2)$  sum rule which is more reliable than  $\Pi_q(q^2)$  sum rule, where  $\Pi_q(q^2)$  and  $\Pi_1(q^2)$  are two invariant functions of the correlator  $\Pi(q^2)$ . We also present the sum rules for the  $K^+p$  and the  $\pi^+\Sigma^+$  multiquark states, and compare to those for the  $\pi^0\Sigma^0$  multiquark state. The mass of the  $\Lambda(1600)$  can be also reproduced in our approach.

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## 1. INTRODUCTION

There have been many works to study properties of the  $\Lambda(1405)$  and its roles in nuclear physics [1–16]. However, its nature is not revealed completely, i.e., an ordinary three quark state or a  $\bar{K}N$  bound state or a mixed state of the previous two possibilities. For a historical review and another references, see [17]. The  $\Lambda(1405)$  has long been considered as a  $\bar{K}N$  bound state [18,19] since it is just below the  $\bar{K}N$  threshold. Thus, it is assumed to be a candidate of hadronic molecules, which are weakly-bound states of two or more hadrons<sup>1</sup>. In this paper we use the QCD sum rule approach to predict a mass of the  $\Lambda(1405)$  considering a multiquark picture.

QCD sum rule [21–23] is one of powerful tools to extract the hadron properties directly from QCD. Applications of this approach to the  $\Lambda(1405)$  were done in Ref. [24–26], respectively. In Ref. [24] it was shown that the five quark operators, which correspond to a baryon and pseudoscalar meson or a baryon and vector meson, have great contribution to the mass of the  $\Lambda(1405)$ . This result is consistent with an analysis from the MIT bag model [27]. On the other hand, Leinweber [25] obtained a good fit to the splitting between the  $\Lambda(1405)$  and the  $\Lambda(1520)$  using a three-quark interpolating field. Recently, as an another approach, Kim and Lee [26] proposed a three-quark interpolating field with a covariant derivative for the  $\Lambda(1405)$  according to the quark configuration of that in the bag model or the non-relativistic quark model.

On the basis of Liu’s result [24] one can assume the  $\Lambda(1405)$  as a  $\bar{K}N$  hadronic molecular state (five-quark state) and investigate the possibility following the same procedures in Ref. [28]. However, in the case of the  $K^-p$  multiquark state there are no exchange diagrams at tree level such as those shown in Ref. [28]. Thus, at tree level it is impossible to determine whether there is a binding effect or not in the  $K^-p$  multiquark state. On the other hand the  $\Lambda(1405)$  is observed in the mass spectrum of the  $\pi\Sigma$  channel ( $I=0$ ), so it would be interesting to get a mass assuming the  $\pi\Sigma$  multiquark state instead of the  $\bar{K}N$  multiquark state. Among three  $\pi\Sigma$  channels, only the  $\pi^0\Sigma^0$  multiquark state has exchange diagrams in contrast to the  $\pi^+\Sigma^-$  and  $\pi^-\Sigma^+$  multiquark states.

In Sec. 2, we present a QCD sum rule prediction of the mass of the  $\Lambda(1405)$  taking into account the  $\pi^0\Sigma^0$  interpolating field, and in Sec. 3 we compare the results of the  $\pi^0\Sigma^0$  multiquark state with those of the  $K^+p$  multiquark state ( $I=1$ ) and the  $\pi^+\Sigma^+$  multiquark state ( $I=2$ ), respectively. We discuss uncertainties in our sum rules and summarize our results in Sec. 4.

## 2. QCD SUM RULE FOR $\pi^0\Sigma^0$ MULTIQUARK

Let’s consider the following correlator:

$$\Pi(q^2) = i \int d^4x e^{iqx} \langle T(J(x)\bar{J}(0)) \rangle, \quad (1)$$

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<sup>1</sup>For a review of hadronic molecules, see [20]

where  $J = \epsilon_{abc}(\bar{u}_e i\gamma^5 u_e - \bar{d}_e i\gamma^5 d_e)([u_a^T C\gamma_\mu s_b]\gamma^5 \gamma^\mu d_c + [d_a^T C\gamma_\mu s_b]\gamma^5 \gamma^\mu u_c)$  corresponds to the interpolating field for the  $\pi^0 \Sigma^0$  multi-quark state.  $u$ ,  $d$  and  $s$  are the up, down and strange quark fields, and  $a, b, c, e$  are color indices.  $T$  denotes the transpose in Dirac space, and  $C$  is the charge conjugation matrix. The OPE side has two structures:

$$\Pi^{OPE}(q^2) = \Pi_q^{OPE}(q^2)\not{A} + \Pi_1^{OPE}(q^2)\mathbf{1}, \quad (2)$$

where

$$\begin{aligned} \Pi_q^{OPE}(q^2) = & -\frac{11}{\pi^8 2^{17} 3^2 5^2 7} q^{10} \ln(-q^2) + \frac{11 m_s^2}{\pi^8 2^{17} 3^2 5} q^8 \ln(-q^2) \\ & - \frac{11 m_s}{\pi^6 2^{13} 3^2 5} \langle \bar{s}s \rangle q^6 \ln(-q^2) - \frac{3}{\pi^4 2^{11}} \langle \bar{q}q \rangle^2 q^4 \ln(-q^2) \\ & + \frac{3 m_s^2}{\pi^4 2^8} \langle \bar{q}q \rangle^2 q^2 \ln(-q^2) - \frac{3 m_s}{\pi^2 2^7} \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle \ln(-q^2) \\ & - \frac{11}{2^3 3^3} \langle \bar{q}q \rangle^4 \frac{1}{q^2}, \end{aligned} \quad (3)$$

and

$$\begin{aligned} \Pi_1^{OPE}(q^2) = & -\frac{11 m_s}{\pi^8 2^{18} 3^2 5^2} q^{10} \ln(-q^2) + \frac{11}{\pi^6 2^{15} 3^2 5} \langle \bar{s}s \rangle q^8 \ln(-q^2) \\ & + \frac{11 m_s^2}{\pi^6 2^{14} 3^2} \langle \bar{s}s \rangle q^6 \ln(-q^2) - \frac{49 m_s}{\pi^4 2^9 3^2} \langle \bar{q}q \rangle^2 q^4 \ln(-q^2) \\ & + \frac{3}{\pi^2 2^6} \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle q^2 \ln(-q^2) - \frac{m_s^2}{\pi^2 2^6 3} (14 \langle \bar{q}q \rangle^3 - 9 \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle) \ln(-q^2) \\ & - \frac{m_s}{2^4 3^3} (44 \langle \bar{q}q \rangle^4 + 3 \langle \bar{q}q \rangle^3 \langle \bar{s}s \rangle) \frac{1}{q^2}. \end{aligned} \quad (4)$$

Here, we let  $m_u = m_d = 0 \neq m_s$  and  $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle \equiv \langle \bar{q}q \rangle \neq \langle \bar{s}s \rangle$ . We neglect the contribution of gluon condensates and concentrate on tree diagrams, and assume the vacuum saturation hypothesis to calculate quark condensates of higher dimensions. Similar calculation was done in Kodama *et al.*'s H-dibaryon sum rules [29]. Note that in  $\Pi_q^{OPE}$  we neglect the term which is proportional to  $m_s^2 \langle \bar{q}q \rangle^4 \frac{1}{q^4}$  in order to keep the same order of power corrections as in  $\Pi_1^{OPE}$ , but its contribution is less than 1 %.

The OPE sides have the following form:

$$\begin{aligned} \Pi_{q,1}^{OPE}(q^2) = & a q^{10} \ln(-q^2) + b q^8 \ln(-q^2) + c q^6 \ln(-q^2) + d q^4 \ln(-q^2) \\ & + e q^2 \ln(-q^2) + f \ln(-q^2) + g \frac{1}{q^2}, \end{aligned} \quad (5)$$

where  $a, b, c, \dots, g$  are constants. Then, we parameterize the phenomenological sides as

$$\begin{aligned} \frac{1}{\pi} \text{Im} \Pi_q^{Phen}(s) = & \lambda^2 \delta(s - m^2) + [-a s^5 - b s^4 - c s^3 - d s^2 - e s - f] \theta(s - s_0), \\ \frac{1}{\pi} \text{Im} \Pi_1^{Phen}(s) = & \lambda^2 m \delta(s - m^2) + [-a s^5 - b s^4 - c s^3 - d s^2 - e s - f] \theta(s - s_0), \end{aligned} \quad (6)$$

where  $m$  is the mass of the  $\pi^0\Sigma^0$  multiquark state and  $s_0$  a continuum threshold for each sum rules.  $\lambda$  is the coupling strength of the interpolating field to the physical  $\Lambda$  (1405) state. After Borel transformation we obtain the mass of the  $\pi^0\Sigma^0$  multiquark state from  $\Pi_q$  and  $\Pi_1$ , respectively. The mass  $m$  is given by

$$\begin{aligned}
m^2 = M^2 \times & \\
& \left\{ -720a(1 - \Sigma_6) - \frac{120b}{M^2}(1 - \Sigma_5) - \frac{24c}{M^4}(1 - \Sigma_4) \right. \\
& \left. - \frac{6d}{M^6}(1 - \Sigma_3) - \frac{2e}{M^8}(1 - \Sigma_2) - \frac{f}{M^{10}}(1 - \Sigma_1) \right\} / \\
& \left\{ -120a(1 - \Sigma_5) - \frac{24b}{M^2}(1 - \Sigma_4) - \frac{6c}{M^4}(1 - \Sigma_3) \right. \\
& \left. - \frac{2d}{M^6}(1 - \Sigma_2) - \frac{e}{M^8}(1 - \Sigma_1) - \frac{f}{M^{10}}(1 - \Sigma_0) - \frac{g}{M^{12}} \right\}, \tag{7}
\end{aligned}$$

where

$$\Sigma_i = \sum_{k=0}^i \frac{s_0^k}{k! (M^2)^k} e^{-\frac{s_0}{M^2}}. \tag{8}$$

In Eq. (7) the continuum contribution is large, so this formula has large uncertainties. We can not find a plateau for the mass of the  $\pi^0\Sigma^0$  multiquark state in the relevant Borel region. Fig. 1 shows the masses from  $\Pi_q$  and  $\Pi_1$  sum rules, where the continuum threshold is taken to be  $s_0 = 2.789 \text{ GeV}^2$  considering the next  $\Lambda$  (1670) particle. The solid line is the mass prediction from  $\Pi_q$  sum rule while the dotted line is that from  $\Pi_1$  sum rule. It seems that  $\Pi_1$  sum rule is more stable than  $\Pi_q$  sum rule. This is consistent with a recent work by Jin and Tang [30]. They showed that the chiral-odd sum rule ( $\Pi_1$  sum rule) is generally more reliable than the chiral-even sum rule ( $\Pi_q$  sum rule) because the positive- and negative-parity excited-state contributions partially cancel each other in  $\Pi_1$  sum rule, but add up in  $\Pi_q$  sum rule. In Fig. 1 there is a plateau for large Borel mass, but it is a trivial result from our crude model on the phenomenological side.

Figs. 2 (a), (b) denote the dependencies of the predicted mass on the s-quark mass and the s-quark condensate. Only the results from  $\Pi_1$  sum rule are shown in the limited Borel region at the same continuum threshold, i.e.,  $s_0 = 2.789 \text{ GeV}^2$ . It seems that the SU(3) symmetry breaking effect is small in our sum rules. On the other hand the mass is rather dependent on the quark condensate as shown in Fig. 2 (c).

First, before getting the mass of the  $\Lambda$  (1405), we can apply the same procedures as in Ref. [28] and determine whether there is a binding effect in the  $\pi^0\Sigma^0$  multiquark state. In the followings we introduce an effective threshold  $s_0$  which will be used to obtain the mass of the  $\Lambda$  (1405). The OPE sides can be rewritten as

$$\Pi_{q,1}^{OPE}(q^2) = N_c^2 (2 \text{ loop-type} + \frac{1}{N_c} \times 1 \text{ loop-type}), \tag{9}$$

where 2 loop-type means the contribution from diagrams of Fig. 3 (a), and 1 loop-type the contribution from diagrams of Fig. 3 (b).  $N_c$  is the number of color, and in Eqs. (3), (4) above we take  $N_c = 3$ . Note that in Figs. 3 (a), (b) we present only some typical diagrams.

Then, we can use the same strategy in Ref. [28]: First, consider 2 loop-type only and vary the continuum threshold  $s_0$  and the Borel interval  $M^2$  in order that the mass should be 1.328 GeV (the sum of a pion and a  $\Sigma$  mass). The Borel interval  $M^2$  is restricted by the following conditions as usual: The lower limit of  $M^2$  is determined as the value at which the power correction is below than 30%. The upper limit is determined as the value where the continuum contribution in the mass prediction is less than 50%. Second, consider all diagrams (1 loop-type + 2 loop-type) and get a new mass  $m'$  at the same  $s_0$  and Borel interval  $M^2$  which are obtained from the first step. Third, compare  $m'$  with 1.328 GeV. If  $m'$  is less than 1.328 GeV, it can be one signature for a molecular-like multiquark state.

Figs. 4 (a), (b) show the dependence of the mass on the Borel mass  $M^2$  in both sum rules, i.e.,  $\Pi_q$  and  $\Pi_1$  sum rule, assuming  $\langle\bar{q}q\rangle = -(0.230 \text{ GeV})^3$ ,  $\langle\bar{s}s\rangle = 0.8 \langle\bar{q}q\rangle$ , and  $m_s = 0.150 \text{ GeV}$ . The solid line is the mass prediction including 2 loop-type diagrams only ( $m$ ), and the dotted line is the mass with all diagrams ( $m'$ ). Because there is no plateau in the valid Borel region for both cases, we determine the mass as an average value in that Borel interval. The results are given in Table 1. It shows that there is a binding effect in  $\Pi_q$  sum rule while a repulsive effect in  $\Pi_1$  sum rule. The binding effect is about 32 MeV in  $\Pi_q$  sum rule. However, in  $\Pi_1$  sum rule the average mass is slightly larger than the  $\pi\Sigma$  threshold. The difference between the average mass and the threshold value is about 3 MeV as shown in the table.

We also present the dependence of  $m'$  on different quark condensates in Table 1. The mass is rarely changed for different quark condensates. The dependencies of  $m'$  on the s-quark mass and the s-quark condensate are given in Tables 2 and 3, respectively. In our approach, if we take another s-quark mass and/or s-quark condensate, then the continuum threshold and/or the Borel interval also should be changed to reproduce the mass  $m$  (1.328 GeV). Thus, the variation of the mass  $m'$  is small.

As we said,  $\Pi_q$  sum rule is less reliable than  $\Pi_1$  sum rule, and if we take a large value of the quark condensate the average mass becomes slightly smaller than the threshold in the case of  $\Pi_1$  sum rule. Hence, at present stage it is difficult to determine whether there is the binding effect in the  $\pi^0\Sigma^0$  multiquark state.

Now, let us determine the mass of the  $\Lambda(1405)$ . Fig. 5 shows the coupling strength  $\lambda^2$  from  $\Pi_q$  and  $\Pi_1$  sum rule at the threshold  $s_0 = 3.122 \text{ GeV}^2$  and  $s_0 = 3.012 \text{ GeV}^2$ , respectively. There is the maximum point in the case of  $\Pi_1$  sum rule while not in the case of  $\Pi_q$  sum rule. Then, we can determine the mass of the  $\Lambda(1405)$  when the coupling strength becomes the maximum value at the threshold  $s_0$  obtained in the previous calculation. In Table 4 the calculated masses are presented for different quark condensates. In Tables 5 and 6 the dependencies of the mass on the s-quark mass and the s-quark condensate are given. The predicted mass 1.419 GeV for  $\langle\bar{q}q\rangle = -(0.230 \text{ GeV})^3$  is similar to the experimental value.

### 3. QCD SUM RULE FOR $K^+P$ AND $\pi^+\Sigma^+$ MULTIQUARK

In this section we apply the previous approach to the  $K^+p$  multiquark state (I=1) and the  $\pi^+\Sigma^+$  multiquark state (I=2) each other. These channels are not exist as a resonance contrary to the  $\pi^0\Sigma^0$  channel. Thus, the results for these multiquark states should be different from those for the  $\pi^0\Sigma^0$  multiquark state. In the case of the  $K^+p$  multiquark state

the OPE sides are given as follows.

$$\begin{aligned}
\Pi_q^{OPE}(q^2) = & -\frac{1}{\pi^8 2^{16} 3^2 5 7} q^{10} \ln(-q^2) + \frac{m_s^2}{\pi^8 2^{16} 3^2} q^8 \ln(-q^2) \\
& + \frac{m_s}{\pi^6 2^{12} 3^2 5} (7\langle \bar{q}q \rangle - 5\langle \bar{s}s \rangle) q^6 \ln(-q^2) \\
& - \frac{1}{\pi^4 2^{10} 3^2} (2\langle \bar{q}q \rangle^2 + 7\langle \bar{q}q \rangle \langle \bar{s}s \rangle) q^4 \ln(-q^2) \\
& + \frac{m_s^2}{\pi^4 2^8 3^2} (4\langle \bar{q}q \rangle^2 - 7\langle \bar{q}q \rangle \langle \bar{s}s \rangle) q^2 \ln(-q^2) \\
& + \frac{m_s}{\pi^2 2^5 3^2} (5\langle \bar{q}q \rangle^3 - \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle) \ln(-q^2) \\
& - \frac{5}{2^2 3^3} \langle \bar{q}q \rangle^3 \langle \bar{s}s \rangle \frac{1}{q^2},
\end{aligned} \tag{10}$$

$$\begin{aligned}
\Pi_1^{OPE}(q^2) = & + \frac{1}{\pi^6 2^{14} 3^2} \langle \bar{q}q \rangle q^8 \ln(-q^2) - \frac{5m_s^2}{\pi^6 2^{12} 3^2} \langle \bar{q}q \rangle q^6 \ln(-q^2) \\
& - \frac{m_s}{\pi^4 2^9 3^2} (4\langle \bar{q}q \rangle^2 - 5\langle \bar{q}q \rangle \langle \bar{s}s \rangle) q^4 \ln(-q^2) \\
& + \frac{1}{\pi^2 2^5 3^2} (7\langle \bar{q}q \rangle^3 + 2\langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle) q^2 \ln(-q^2) \\
& - \frac{m_s^2}{\pi^2 2^4 3^2} (7\langle \bar{q}q \rangle^3 - \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle) \ln(-q^2) \\
& - \frac{m_s}{2^3 3^3} (20\langle \bar{q}q \rangle^4 - 7\langle \bar{q}q \rangle^3 \langle \bar{s}s \rangle) \frac{1}{q^2},
\end{aligned} \tag{11}$$

and for the  $\pi^+ \Sigma^+$  multiquark state we get

$$\begin{aligned}
\Pi_q^{OPE}(q^2) = & -\frac{1}{\pi^8 2^{16} 3^2 5 7} q^{10} \ln(-q^2) + \frac{m_s^2}{\pi^8 2^{16} 3^2} q^8 \ln(-q^2) \\
& - \frac{m_s}{\pi^6 2^{12} 3^2} \langle \bar{s}s \rangle q^6 \ln(-q^2) - \frac{1}{\pi^4 2^{10}} \langle \bar{q}q \rangle^2 q^4 \ln(-q^2) \\
& + \frac{m_s^2}{\pi^4 2^7} \langle \bar{q}q \rangle^2 q^2 \ln(-q^2) - \frac{m_s}{\pi^2 2^6} \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle \ln(-q^2) \\
& - \frac{5}{2^2 3^3} \langle \bar{q}q \rangle^4 \frac{1}{q^2},
\end{aligned} \tag{12}$$

and

$$\begin{aligned}
\Pi_1^{OPE}(q^2) = & -\frac{m_s}{\pi^8 2^{17} 3^2 5} q^{10} \ln(-q^2) + \frac{1}{\pi^6 2^{14} 3^2} \langle \bar{s}s \rangle q^8 \ln(-q^2) \\
& + \frac{5 m_s^2}{\pi^6 2^{13} 3^2} \langle \bar{s}s \rangle q^6 \ln(-q^2) - \frac{m_s}{\pi^4 2^8} \langle \bar{q}q \rangle^2 q^4 \ln(-q^2) \\
& + \frac{1}{\pi^2 2^5} \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle q^2 \ln(-q^2) + \frac{m_s^2}{\pi^2 2^5} \langle \bar{q}q \rangle^2 \langle \bar{s}s \rangle \ln(-q^2) \\
& - \frac{5 m_s}{2 3^3} \langle \bar{q}q \rangle^4 \frac{1}{q^2},
\end{aligned} \tag{13}$$

where we take  $N_c = 3$  as in the previous  $\pi^0\Sigma^0$  multiquark sum rules (hereafter  $\pi^0\Sigma^0$  sum rules).

Comparing Eqs. (12) and (13) to Eqs. (10) and (11) each other one can easily find that the formulas are exactly the same in the SU(3) symmetric limit. Then, two SU(3) symmetry breaking parameters ( $m_s$  and  $\gamma \equiv \frac{\langle \bar{s}s \rangle}{\langle \bar{q}q \rangle} - 1$ ) give different characteristics between the  $K^+p$  and the  $\pi^+\Sigma^+$  sum rules. Fig. 6 and Fig. 7 are the mass predictions, where we use  $\langle \bar{q}q \rangle = -(0.230 \text{ GeV})^3$ ,  $\langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle$ ,  $m_s = 0.150 \text{ GeV}$ . We let  $m = m_{K^+} + m_p = 1.435 \text{ GeV}$  and  $m = m_{\pi^+} + m_{\Sigma^+} = 1.329 \text{ GeV}$  for the  $K^+p$  sum rules and the  $\pi^+\Sigma^+$  sum rules, respectively. They show the same pattern for each sum rules. In Tables 7 and 8 we present the average masses in the valid Borel interval. It shows that there is a binding effect in  $\Pi_q$  sum rule for both multiquark states. The binding effect is about 60 MeV and 100 MeV for the  $K^+p$  and the  $\pi^+\Sigma^+$  multiquark state, respectively. On the other hand, in  $\Pi_1$  sum rule there is a repulsive effect for both states although the magnitude is very small in our approach, and this is the same as the experimental result [31]. Thus, also in these two cases  $\Pi_1$  sum rule seems more reliable than  $\Pi_q$  sum rule.

We can apply the previous method to get the mass of the  $K^-p$  multiquark state. The  $K^-p$  sum rule is the same as the  $K^+p$  sum rule when only 2-loop diagrams are considered. Using the same threshold in Table 7, i.e.,  $s_0 = 3.852 \text{ GeV}^2$  we get 1.589 GeV from  $\Pi_1$  sum rule. We also obtain the mass of the  $\pi^-\Sigma^+$  multiquark state from the  $\pi^+\Sigma^+$  sum rule. The predicted mass at the same  $s_0 = 3.852 \text{ GeV}^2$  is 1.606 GeV. In the case of the  $\pi^0\Sigma^0$  sum rule we get 1.625 GeV at this threshold. It is interesting to note that these values are very similar to each other, and close to that of the  $\Lambda(1600)$  which can decay to both  $\bar{K}N$  and  $\pi\Sigma$  channels [17].

Similarly, taking into account the threshold for the  $\pi\Sigma$  sum rule we can get the mass of the  $K^-p$  multiquark state. The masses are 1.387 GeV and 1.412 GeV for  $s_0 = 3.012 \text{ GeV}^2$  and  $s_0 = 3.112 \text{ GeV}^2$ , respectively. They reproduce the  $\Lambda(1405)$  mass. On the other hand, the mass of the  $\pi^-\Sigma^+$  multiquark state at the threshold  $s_0 = 3.112 \text{ GeV}^2$  is 1.426 GeV.

#### 4. DISCUSSION

Let us discuss uncertainties in our sum rules. First, most uncertainties come from neglecting the contribution of other dimensional operators (e.g., gluon condensates) on the OPE side. As we said previously, there is only one power correction term in our sum rules, thus the results are very sensitive to the choice of the continuum threshold  $s_0$ . Second, one of uncertainties results from assuming the vacuum saturation hypothesis to calculate quark condensates of higher dimensions. In  $\Pi_1$  sum rule the dominant operator has the form of  $\langle \bar{q}q \rangle^3$  while  $\langle \bar{q}q \rangle^2$  in  $\Pi_q$  sum rule. Thus, the uncertainty contributes to each sum rule in a different manner. Last, in the previous  $\pi\Sigma$  sum rules we only consider the  $\pi^0\Sigma^0$  channel. It would be necessary to obtain the sum rules for the full basis, i.e.,  $\pi^+\Sigma^- + \pi^0\Sigma^0 + \pi^-\Sigma^+$  multiquark state. Full quantitative analysis, however, would require all the above corrections and is beyond the scope of this paper.

In summary, the mass of the  $\Lambda(1405)$  is predicted using the  $\pi^0\Sigma^0$  multiquark interpolating field. The predicted mass from  $\Pi_1$  sum rule (the chiral-odd sum rule) is about 1.419 GeV. The mass sum rules for the  $K^+p$  and the  $\pi^+\Sigma^+$  multiquark state are also presented, and

compared to those for the  $\pi^0\Sigma^0$  multiquark state. It is necessary to investigate the problem further both theoretically and experimentally to determine the structure of the  $\Lambda(1405)$ . On the other hand, it would be interesting to calculate the mass of the  $\Lambda(1405)$  with the baryon and vector meson (e.g.,  $\Sigma\rho$ ) interpolating field which was proposed in [27,24].

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# TABLES

TABLE 1.  $\pi^0 \Sigma^0$  multiquark state ( $\langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle$ ,  $m_s = 0.150$  GeV)

	quark condensate ( $\text{GeV}^3$ )	$s_0$ ( $\text{GeV}^2$ )	$M^2$ ( $\text{GeV}^2$ )	$m'$ (GeV)
$\Pi_q(q^2)$	$-(0.210)^3$	3.005	0.82 – 1.68	1.303
	$-(0.230)^3$	3.122	0.98 – 1.80	1.296
	$-(0.250)^3$	3.268	1.18 – 1.90	1.289
$\Pi_1(q^2)$	$-(0.210)^3$	3.015	0.98 – 1.80	1.333
	$-(0.230)^3$	3.012	1.18 – 1.90	1.331
	$-(0.250)^3$	3.008	1.40 – 2.00	1.330

TABLE 2.  $\pi^0 \Sigma^0$  multiquark state ( $\langle \bar{q}q \rangle = -(0.230 \text{ GeV})^3$ ,  $\langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle$ )

	s-quark mass (GeV)	$s_0$ ( $\text{GeV}^2$ )	$M^2$ ( $\text{GeV}^2$ )	$m'$ (GeV)
$\Pi_q(q^2)$	0.120	3.143	1.00 – 1.80	1.299
	0.180	3.094	0.96 – 1.80	1.291
$\Pi_1(q^2)$	0.120	3.030	1.20 – 1.90	1.331
	0.180	2.996	1.16 – 1.90	1.331

TABLE 3.  $\pi^0 \Sigma^0$  multiquark state ( $\langle \bar{q}q \rangle = -(0.230 \text{ GeV})^3$ ,  $m_s = 0.150$  GeV)

	s-quark condensate	$s_0$ ( $\text{GeV}^2$ )	$M^2$ ( $\text{GeV}^2$ )	$m'$ (GeV)
$\Pi_q(q^2)$	0.6 $\langle \bar{q}q \rangle$	3.146	1.00 – 1.80	1.299
	1.0 $\langle \bar{q}q \rangle$	3.095	0.96 – 1.80	1.293
$\Pi_1(q^2)$	0.6 $\langle \bar{q}q \rangle$	2.984	1.16 – 1.90	1.331
	1.0 $\langle \bar{q}q \rangle$	3.030	1.20 – 1.90	1.331

TABLE 4. Mass of the  $\pi^0 \Sigma^0$  multiquark state ( $\langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle$ ,  $m_s = 0.150$  GeV)

	quark condensate ( $\text{GeV}^3$ )	$s_0$ ( $\text{GeV}^2$ )	$m$ (GeV)
$\Pi_1(q^2)$	$-(0.210)^3$	3.015	1.434
	$-(0.230)^3$	3.012	1.419
	$-(0.250)^3$	3.008	1.404

TABLE 5. Mass of the  $\pi^0 \Sigma^0$  multiquark state ( $\langle \bar{q}q \rangle = -(0.230 \text{ GeV})^3$ ,  $\langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle$ )

	s-quark mass (GeV)	$s_0$ (GeV <sup>2</sup> )	$m$ (GeV)
$\Pi_1(q^2)$	0.120	3.030	1.419
	0.180	2.996	1.419

TABLE 6. Mass of the  $\pi^0 \Sigma^0$  multiquark state ( $\langle \bar{q}q \rangle = -(0.230 \text{ GeV})^3$ ,  $m_s = 0.150 \text{ GeV}$ )

	s-quark condensate	$s_0$ (GeV <sup>2</sup> )	$m$ (GeV)
$\Pi_1(q^2)$	0.6 $\langle \bar{q}q \rangle$	2.984	1.417
	1.0 $\langle \bar{q}q \rangle$	3.030	1.419

TABLE 7.  $K^+ p$  multiquark state ( $\langle \bar{q}q \rangle = -(0.230 \text{ GeV})^3$ ,  $\langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle$ ,  $m_s = 0.150 \text{ GeV}$ )

	$s_0$ (GeV <sup>2</sup> )	$M^2$ (GeV <sup>2</sup> )	$m'$ (GeV)
$\Pi_q(q^2)$	3.646	1.06 – 2.00	1.372
$\Pi_1(q^2)$	3.852	0.94 – 2.22	1.440

TABLE 8.  $\pi^+ \Sigma^+$  multiquark state ( $\langle \bar{q}q \rangle = -(0.230 \text{ GeV})^3$ ,  $\langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle$ ,  $m_s = 0.150 \text{ GeV}$ )

	$s_0$ (GeV <sup>2</sup> )	$M^2$ (GeV <sup>2</sup> )	$m'$ (GeV)
$\Pi_q(q^2)$	3.126	0.98 – 1.80	1.239
$\Pi_1(q^2)$	3.112	1.20 – 1.88	1.330

## FIGURES

FIG. 1. Mass of the  $\pi^0\Sigma^0$  multiquark state at the continuum threshold  $s_0 = 2.789 \text{ GeV}^2$ . The solid line is the predicted mass from  $\Pi_q$  sum rule, and the dotted line is that from  $\Pi_1$  sum rule.

FIG. 2. Dependence of the mass of the  $\pi^0\Sigma^0$  multiquark state from  $\Pi_1$  sum rule on (a) strange quark mass (b) strange quark condensate (c) quark condensate .

FIG. 3. Diagrams. Solid lines are the quark propagators. (a) 2 loop-type (b) 1 loop-type .

FIG. 4. Mass of the  $\pi^0\Sigma^0$  multiquark state in the valid Borel region.  $m$  is the mass with 2 loop-type diagrams, and  $m'$  is the mass with all diagrams (1 loop-type + 2 loop-type). (a)  $\Pi_q$  sum rule (b)  $\Pi_1$  sum rule.

FIG. 5. Coupling strength  $\lambda^2$  from  $\Pi_q$  and  $\Pi_1$  sum rule.

FIG. 6. Mass of the  $K^+p$  multiquark state in the valid Borel region. The same as Fig. 4.

FIG. 7. Mass of the  $\pi^+\Sigma^+$  multiquark state in the valid Borel region. The same as Fig. 4.





























